

Importance of the Doppler Effect to the Determination of the Deuteron Binding Energy

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The deuteron binding energy extracted from the reaction $^1H(n, \gamma)^2H$ is reviewed with the exact relativistic formula, where the initial kinetic energy and the Doppler effect are taken into account. We find that the negligible initial kinetic energy of the neutron could cause a significant uncertainty which is beyond the errors available up to now. Therefore, we suggest an experiment which should include the detailed informations about the initial kinetic energy and the detection angle. It could reduce discrepancies among the recently reported values about the deuteron binding energy and pin down the uncertainty due to the Doppler broadening of γ ray.

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The deuteron binding energy is one of the most important physical quantities in nuclear physics. This plays a central role in the determination of the neutron mass, and provides a critical reference energy for the determination of high-energy γ rays measured on the atomic-mass scale [1]. There are several articles to have measured and improved the deuteron binding energy more precisely than ever [1–7]. Some of them have reported their values as precisely as up to the order of several electron volts [1,2,4,6]. Usually, the deuteron binding energy is determined by adding the recoil energy of the deuteron to the measured energy of the γ ray which is emitted from the neutron capture reaction, $^1H(n, \gamma)^2H$. In the conventional non-relativistic treatment, there seems to be nothing to correct any more, because the initial kinetic energy of the neutron is assumed to be so small that it can be neglected safely. Actually, the kinetic energy of the neutron used in Ref. [1] is 0.056 eV and that of the thermalized neutron from the neutron source in Ref. [2,4,5] is assumed to be 0.025 eV at room temperature.

However, if the two body collision process is treated relativistically, we demonstrate that such a small kinetic energy of the neutron causes a considerable uncertainty in the deuteron binding energy because of the Doppler effect. Even the very small neutron kinetic energy gives rise to a significant velocity of the n-p system in the laboratory frame. As a result, this moving source of the γ ray can be an origin for the Doppler effect. Naturally this Doppler effect depends on the angle of the γ ray detector with respect to the velocity of moving source. Therefore, the measured γ ray energy is expected to have an unavoidable uncertainty originating from this angle dependence. As will be shown later on, our estimation for the uncertainty shows about 25 eV maximally for the neutron initial kinetic energy of 0.056 eV and 14.0 eV for that of 0.025 eV, respectively. These values are so large compared to the reported error 2.3 eV [1], which comes from the γ ray detector itself.

Let us suppose that two particles are initially at rest within an appropriate interaction distance, and they come together to form one body system by emitting a photon. In this situation, the center of momentum frame [12], which is an exact nomenclature in relativistic kinematics but used as the center of mass frame hereafter, coincides with the laboratory frame. From the energy and momentum conservation, we get the following relation

$$p_1 + p_2 = q + k, \quad (1)$$

where p_i 's are four momenta of the initial two particles while q and k are those of the resultant composite particle and the photon, respectively. The time component of the momentum accounts for the energy conservation, and its space components stand for the momentum conservation. The square of Eq. (1) is Lorentz invariant:

$$(p_1 + p_2)^2 = (q + k)^2. \quad (2)$$

In the case of the initial particles at rest, this gives the following equation in the center of mass frame

$$(m_1 + m_2)^2 = M^2 + 2\omega^2 + 2\omega\sqrt{M^2 + \omega^2}, \quad (3)$$

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where M and m_i are masses of the composite and the initial particles, respectively, and ω is the energy of the emitted photon. Solving for ω , we obtain the exact equation for the energy of the emitted photon in terms of the masses of the initial and the final particles:

$$\begin{aligned}\omega &= \frac{(m_1 + m_2 + M)(m_1 + m_2 - M)}{2(m_1 + m_2)} \\ &= \frac{(E_b + 2M)E_b}{2(E_b + M)},\end{aligned}\quad (4)$$

where the binding energy E_b is defined as the mass difference between the initial particles and the final product $E_b = m_1 + m_2 - M$. From Eq. (4), we get the binding energy in terms of the produced mass and the energy of the emitted photon:

$$\begin{aligned}E_b &= \omega + \sqrt{\omega^2 + M^2} - M \\ &\cong \omega + \frac{\omega^2}{2M},\end{aligned}\quad (5)$$

where the last expression shows the non-relativistic approximation which is used to determine the binding energy of the deuteron so far [1–7].

In the case that the incident particle has a kinetic energy and comes to the target at rest, the laboratory frame differs from the center of mass frame. Using the Lorentz invariance of the square of four momentum, we obtain the following equation

$$(p_1 + p_2)_{lab}^2 = (p_1 + p_2)_{c.m.}^2 = (q + k)_{c.m.}^2, \quad (6)$$

which is calculated as

$$m_1^2 + m_2^2 + 2m_2E_{1l} = M^2 + 2\omega^2 + 2\omega\sqrt{M^2 + \omega^2}. \quad (7)$$

Notice that ω is the energy of the photon emitted in the center of mass frame and is given by

$$\omega = \frac{m_1^2 + m_2^2 - M^2 + 2m_2E_{1l}}{2\sqrt{m_1^2 + m_2^2 + 2m_2E_{1l}}}. \quad (8)$$

Here the velocity of the center of mass frame is calculated as

$$v_{c.m.} = \frac{|\mathbf{p}_{1l}|}{m_2 + E_{1l}}. \quad (9)$$

The above energy of the photon should be transformed to that of the laboratory frame in which the observation is carried out. This transformation is nothing but the Doppler effect and is given, in relativistic theory, by

$$\omega_l = \omega \frac{\sqrt{1 - v_{c.m.}^2}}{1 - \mathbf{n} \cdot \mathbf{v}_{c.m.}}, \quad (10)$$

where \mathbf{n} is the unit vector in the direction of the photon beam. Thus, we obtain the energy of the photon measured in the laboratory frame:

$$\begin{aligned}\omega_l &= \omega \frac{\sqrt{1 - v_{c.m.}^2}}{1 - v_{c.m.} \cos \theta} \\ &= \frac{m_1^2 + m_2^2 - M^2 + 2m_2E_{1l}}{2(m_2 + E_{1l} - |\mathbf{p}_{1l}| \cos \theta)} \\ &= \frac{(E_b + 2M)E_b + 2m_2K_1}{2(E_b + M + K_1 - |\mathbf{p}_{1l}| \cos \theta)},\end{aligned}\quad (11)$$

where the kinetic energy is defined by $K_1 = E_1 - m_1$. It should be noted that ω_l depends on θ , if the neutron beam direction is fixed. Alternatively, the same expression can be derived directly from Eq. (2) in the laboratory frame. We consider here three special cases for easy understanding as explained in Ref. [13]. The first case, $\theta = \frac{\pi}{2}$, which represents the transverse Doppler effect, is when we measure the frequency of the photon emitting perpendicular to

the incident beam. The second case, $\theta = \pi$, which corresponds to the longitudinal Doppler effect in which the source is receding, is when we measure the photon emitting anti-parallel to the incident beam. The last case, $\theta = 0$, which is also longitudinal but the source is approaching, is when we measure the photon emitting parallel to the incident beam. Solving for the binding energy, we obtain the following equation for the binding energy without any approximation

$$E_b = \omega_l - M + \sqrt{M^2 + \omega_l^2 - 2(m_2 - \omega_l)K_1 - 2\omega_l|\mathbf{p}_{1l}|\cos\theta}, \quad (12)$$

where $|\mathbf{p}_{1l}| = \sqrt{K_1(K_1 + 2m_1)}$. This equation, which can be reduced to Eq. (5) as the initial kinetic energy K_1 goes to zero, contains the initial kinetic energy of the neutrons and the detection angle. Both of them were not considered in the non-relativistic analysis of the experiment of $^1H(n, \gamma)^2H$ reaction [1].

In the following, we show the detailed numbers coming from the above θ dependence and compare to the reported error. The measured wavelength of the γ ray emitted from the n-p capture has been reported as [1]

$$\lambda_{np} = 5.576\ 698\ 8\ (55) \times 10^{-13}m. \quad (13)$$

The error in parenthesis comes from the Bragg angle and the lattice-spacing of crystal which are related with the equation $n\lambda = 2d \sin \theta_{Bragg}$. Using the conversion constant $\hbar c$, we obtain the energy of the photon:

$$\omega = \frac{2\pi\hbar c}{\lambda_{np}} = 2.223\ 255\ 2\ (23)\text{MeV}. \quad (14)$$

The physical constants used in this letter are [8]

$$\begin{aligned} \hbar c &= 197.327\ 053\ (59)\text{MeV fm}, \\ \text{deuteron mass } M &= 1875.613\ 39\ (57)\text{MeV}, \\ \text{neutron mass } m_1 &= 939.565\ 63\ (28)\text{MeV}, \\ \text{proton mass } m_2 &= 938.272\ 31\ (28)\text{MeV}. \end{aligned} \quad (15)$$

Since a deuteron mass M in Eq. (12) causes an uncertainty in summing the significant figures, we expanded the equation in order to visualize the cancellation of the uncertainty due to the deuteron mass as follows

$$\begin{aligned} E_b &\cong \omega_l + \frac{\omega_l^2}{2M} - \frac{(m_2 - \omega_l)K_1}{M} - \frac{\omega_l\sqrt{K_1(K_1 + 2m_1)}}{M}\cos\theta \\ &= 2.223\ 255\ 2\ (23) + 0.001\ 317\ 7 - 0.000\ 000\ 03 - 0.000\ 012\ 2\cos\theta\ \text{MeV}. \end{aligned} \quad (16)$$

In this result, the second term is the deuteron recoil effect, the third term comes from the initial kinetic energy itself and the fourth term stands for the Doppler effect due to the moving source of the center of mass frame. Since the authors of Ref. [1] have used the neutron flux of which distribution is approximately Maxwellian with a peak at 1.2 Å (= 0.056 eV), we calculate the binding energies for the three cases of $\theta = 0, \frac{\pi}{2}, \pi$:

$$\begin{aligned} E_b &= 2.224\ 560\ 1\ (23)\ \text{MeV for } \cos\theta = 1, \\ E_b &= 2.224\ 573\ 2\ (23)\ \text{MeV for } \cos\theta = 0, \\ E_b &= 2.224\ 585\ 2\ (23)\ \text{MeV for } \cos\theta = -1. \end{aligned} \quad (17)$$

This Doppler effect will be reflected as a line width, a line shift of γ ray or both of them in the experiments. As we know from the above numerical calculations, the Doppler effect due to the initial kinetic energy of the neutron causes an uncertainty by around 25 eV, which is completely out of the range of the above error 2.3 eV. This error may be the best value of the present measurement technique of the γ ray in that energy scale. Consequently, it is inescapable to take the uncertainty due to this Doppler effects into account in the analysis of the $^1H(n, \gamma)^2H$ experiments.

In the actual experiment using the thermalized neutron, however, the dependence of the angle is not detected, but manifests itself as a line width in γ ray spectrum because of the following reason. Since the incident neutrons may be distributed at random and isotropic in this experiment, we can assume that they are uniformly distributed over the whole solid angle 4π with respect to the direction of the detected photons. Therefore the γ ray detector fixed at the given angle sees all γ rays triggered by the incoming neutrons in the whole angle range, i.e., from the angle $\theta = 0$ to $\theta = \pi$ in Eq. (16). Among those neutrons, the ones from the angle $\theta = \frac{\pi}{2}$ are most probable because of azimuthal symmetry, that is, the number of the incoming neutrons is proportional to $2\pi \sin\theta d\theta$. Consequently, we estimate the width of the spectrum of the emitted photons from Eq. (11) as follows

$$\begin{aligned}
\Delta\omega_l &= \sqrt{\{\omega'_l(K_0, \cos\theta_0)\}^2\Delta K^2 + \{\omega'_l(K_0, \cos\theta_0)\}^2\Delta \cos\theta^2} \\
&= \sqrt{0.5^2\Delta K^2 + 12.2^2\Delta \cos\theta^2} \text{ eV} \\
&= 21.1 \text{ eV, for } \Delta \cos\theta = \sqrt{3} \text{ and } \Delta K = 0.005 \text{ eV,}
\end{aligned} \tag{18}$$

where we expanded it at $K_0 = 0.056$ eV and $\cos\theta_0 = 0$. For the initial neutron kinetic energy of 0.025 eV, the line width is expected to be 14.0 eV. The above numbers are surprisingly large compared to the error from the crystal spectrometer. Here one can expect another line width coming from the uncertainty of the kinetic energy of the incident neutrons, expressed by ΔK in Eq. (18). But it is very small enough to be neglected as calculated above and pointed out in Ref. [1]. Thus, from the above relation, the broadening of the γ ray is mainly due to the Doppler effect. However most of Ge detectors or the other γ ray detectors may not resolve the broadened spectrum, so the crystal spectrometer is used for precise measurements by determining the Bragg angle as in Ref. [1].

In this experimental situation, the broadened γ ray causes the uncertainties in the measurement of the Bragg angle and also in the spectrums of the γ ray detector which determines the peak positions. The authors of Ref. [1] have measured the 52 Bragg angles between the two centers of the peaks of the broadened spectrums. The small error of 2.3 eV, which comes from the statistical error in the data of the 52 Bragg angles and the lattice-spacing of crystal, is much less than the calculated line width of 14.0 - 21.1 eV. The reason of such a small error may be conjectured that the standard error is obtained from the standard deviation by dividing it by the square root of the number of measurements and the errors in the center positions of the broadened spectrums are neglected. So the standard deviation, which is simply calculated as $2.3 \times \sqrt{52} = 16.6$ eV, is compatible with our calculated line width. Therefore if we reduce the line width of the γ ray by selecting the special angle $\theta = \frac{\pi}{2}$, the measurement of the Bragg angle has much smaller error and is much more convincible though it is difficult to reduce the uncertainty of the γ ray detector itself. Actually, the standard deviation of the measured Bragg angle 1.4×10^{-8} rad in Ref. [1] is larger than the sensitivity and accuracy of $\sim 10^{-4}$ arcsec ($\leq 10^{-9}$ rad) of the Michelson interferometer, so that it can be reduced up to the sensitivity.

Table I shows the update values of the deuteron binding energy. Among them, only two articles [2,3] have shown their experimental geometry. So, we discuss these experimental geometry on a possible Doppler effect. Since these experiments seems to be carried out at room temperature, we assume that the kinetic energy of the thermal neutrons is 0.025 eV because there are no more informations on the kinetic energy of the incident neutron. In the experiment of Ref. [2], the incident neutrons seem to be well-thermalized and the neutron source is located to the side of the paraffin sample. The neutrons moving perpendicular to the direction of the detected photons are captured dominantly as the previous consideration, and the width of the detected photons is calculated as 14.0 eV with using Eq. (18). If the paraffin sample is more thin and has larger diameter than that of this experiment and exposed to the thermalized neutrons moving perpendicular to the detected photon by shielding the sample except the round surface of the sample cylinder, one can reduce $\Delta \cos\theta$ and the Doppler broadening of the spectrum of the photon.

In the experiment of Ref. [3], the situation is so complicated to treat incident neutrons as well-thermalized ones. There is a possibility that many fast neutrons can be captured through thermalization in the paraffin sample, because the source emits much higher energy neutrons than the thermal neutrons. Even though it is difficult to know what energy of the neutrons contribute to the capture process dominantly, it is possible to compare this experimental result with others in the point of view of the Doppler shift. In Refs. [2,5], the authors have modified the binding energy of Ref. [3] on the basis of the same standards of the calibration energies of Refs. [9,10] as 2.224 628 (15) MeV, and have compared it with their measured ones. The corrected value is shifted by 53 eV with respect to the value of Ref. [2] and 64 eV with respect to the value of Ref. [5]. These shifts occur, when the kinetic energy of the dominant incident neutron is more than at least 1.104 eV under the assumption that the incident neutrons are parallel to the direction of the detected photons according to our result. The authors of Ref. [3] have also revised their value as 2.224 568 (8) MeV on the basis of the standards of Refs. [9,11] in Ref. [4]. In the point of view of our result, this value can not be understood easily, on the contrary, this discrepancy is attributed to the problem of the calibration using a common system of standard energies. Table II shows other similar experiments, in which it is more important to consider the Doppler effect in the case of a triton.

The direction of motion of the thermal neutrons is distributed at random in the previous experiments [1-7], and Doppler broadening contributes in full to the observed line width. Selecting a certain angle of the incident neutron with respect to the emitted photon will lead to a reduced line width and should therefore make it possible to obtain even more precise results. Such an experiment would be very challenging. A possible geometry could make use of a target sample near the core of a reactor with appropriate shielding from thermal neutrons. Using the difference of the penetration lengths for neutrons and photons, shielding the target should not cause much loss in flux of the incident neutron beam. As shown by the detailed numbers, it is indispensable to reduce the Doppler broadening for a more

precise measurement of the gamma-ray wavelength and the determination of the deuteron binding energy. Careful geometric considerations can also explain the discrepancies among the values of the deuteron binding energy reported in Refs. [1–7], if calibration problems are properly accounted for.

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Reference	Binding energy (eV)
Greene <i>et al.</i> [1]	2 224 589.0 (2.2)
Van der Leun <i>et al.</i> [2]	2 224 575 (9)
Vylov <i>et al.</i> [3]	2 224 572 (40)
Vylov <i>et al.</i> [4]	2 224 568 (8)
Greenwood <i>et al.</i> [5]	2 224 564 (17)
Adam <i>et al</i> [6]	2 224 574 (9)
Michael <i>et al.</i> [7]	2 224 579 (13)

TABLE I. The recently reported values of the deuteron binding energy. The deviations among the values show up at 10 eV order, which is just the same order as the Doppler effects contribute (see Eq. (17)). However notice that some of them have the error of several electron volts order, which is smaller than the Doppler effects.

(n, γ) reaction product nucleus (MeV)	γ -ray energy (MeV)	Binding energy (MeV)	Doppler shift
3 T	6.250 316	6.257 268 (24)	15.2 eV cos θ
13 C	4.949 319	4. 946 329 (24)	2.8 eV cos θ
14 C	8.173 922	8.176 483 (40)	4.2 eV cos θ
15 N	10.829 101	10.833 297 (38)	5.3 eV cos θ

TABLE II. These are other nucleus binding energies from (n, γ) reactions in Ref. [5] and the possible Doppler shifts according to the angle of the incident neutrons with respect to the direction of the detected photons, where we assume that the kinetic energy of thermal neutrons is 0.025 eV.